Large deformation behavior of compliant mechanisms

Article · January 2001

Sridhar Kota
University of Michigan

Noboru Kikuchi
University of Michigan, Toyota Central R&D Labs

Proceedings of DETC'01
ASME 2001 Design Engineering Technical Conference
and Computers and Information in Engineering Conference
Pittsburgh, PA, September 9-12, 2001

DETC2001/DAC-21084

LARGE DEFORMATION BEHAVIOR OF COMPLIANT MECHANISMS

Jinyong Joo
Graduate Student Research Assistant
Department of ME
The University of Michigan
Ann Arbor, MI 48109
jinyong@engin.umich.edu

Sridhar Kota
Professor
Department of ME
The University of Michigan
Ann Arbor, MI 48109
kota@umich.edu

Noboru Kikuchi
Professor
Department of ME
The University of Michigan
Ann Arbor, MI 48109
kikuchi@umich.edu

ABSTRACT

This paper presents a non-linear formulation for size and shape[*] optimization of compliant mechanisms using tapered beam elements. Designs based on linear and nonlinear formulations are compared using a stroke amplification mechanism example. Also, the scaling effect of the compliant mechanism is investigated.

INTRODUCTION

The design of compliant mechanisms involves two steps: (i) Topological synthesis which provides qualitative solutions to satisfy prescribed force-displacements kinematic requirements at the input and output. (ii) Size and shape optimization to determine exact dimensions by taking into consideration performance requirements such as energy efficiency, mechanical advantage or geometrical advantage, and constraints such as permissible stresses and strains. In topology design methodology, researchers [1,2,3,4] consider nonlinear effect because compliant mechanisms are generally operating in large deformation ranges. Joo et al [1] compare topology designs for using linear and nonlinear synthesis methods. Designs based on nonlinear synthesis seem to distribute stresses more uniformly throughout the whole mechanism. Pedersen et al. [2] showed that the increase in output performance of nonlinear synthesis results is as high as 150 % compared to linear synthesis results. In addition, they showed a method for synthesis of path-generating compliant mechanisms using nonlinear formulation. Saxena and Ananthasuresh [3], and Bruns and Tortorelli [4] examined the topology synthesis considering geometrical nonlinearity.

Starting with a known topology, size and shape design scheme is applied to consider quantitative requirement such as allowable stress, geometrical advantage and so on. Hetrick and Kota [5] performed size and shape optimization using energy formulation only with parametric beam elements. However, this method can be applied to designs operating within a small deformation range because it is based on linear theory. Nonlinear size and shape synthesis together with nonlinear topology synthesis is needed to design compliant mechanisms that are subjected to large deformations.

TAPERED BEAM ELEMENTS

Hetrick and Kota [5] used parametric finite beam models to describe compliant links. This technique allows stress constrains to limit the maximum stress in the mechanism [5]. The drawback of this method is that the shapes of the resulting compliant links are not smooth (Figure 1a), due to sudden changes in cross-sections. Tapered beam elements provide a smooth change in cross-section and require fewer (three) design variables per link (Figure 1b).

[*] Size refers to the cross section area of each link and shape (or geometry) refers to the location of nodes.

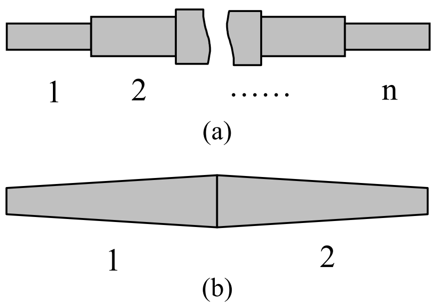


Fig. 1 Parametric beam element and tapered beam element

PROBLEM SPECIFICATION FOR COMPLIANT MECHANISM DESIGN

The objective of this work is to synthesize the size and shape of a compliant mechanism that is intended to undergo large deformation. Therefore, nonlinear analysis methods are employed in the optimization scheme to derive a feasible size and shape. The synthesis procedure considers desired input/output displacement, external loads acting against the mechanism output, allowable stress, geometrical advantage and space constraints.

Figure 2 shows a typical problem specification. The input displacement is applied at point i and the output motion is desired at point o. The external load resisting the output motion is represented by a spring of stiffness Ks at the output port o. The cross sectional areas of individual tapered links at each end and in the middle constitute the design variables to be investigated so as to arrive at optimum size of links (Figure 2b). Additionally, the optimization procedure relocates nodes within prescribed boundaries and thereby arrives at an optimum shape (topology remains unaltered). For example, in Figure 2a, during the optimization procedure node o is allowed to wander within the rectangular region ‘abcd’.

OBJECTIVE FUNCTION FORMULATION

The objective function, defined as maximizing the ratio of output displacement over input displacement (geometrical advantage), is employed for the design of compliant mechanisms. Joo et al. (2000) reported a similar objective function for topology optimization except an additional term for minimizing the strain energy. Prescribed displacement is applied as input rather than force as input to avoid infeasible solution [6]. The drawback of using displacement input is that the input force may be very large. Hence, an input force constraint is added to obtain solutions with forces within an acceptable range. Also, a buckling constraint is added to avoid buckling under compression load. The complete optimization problem is described as follows:

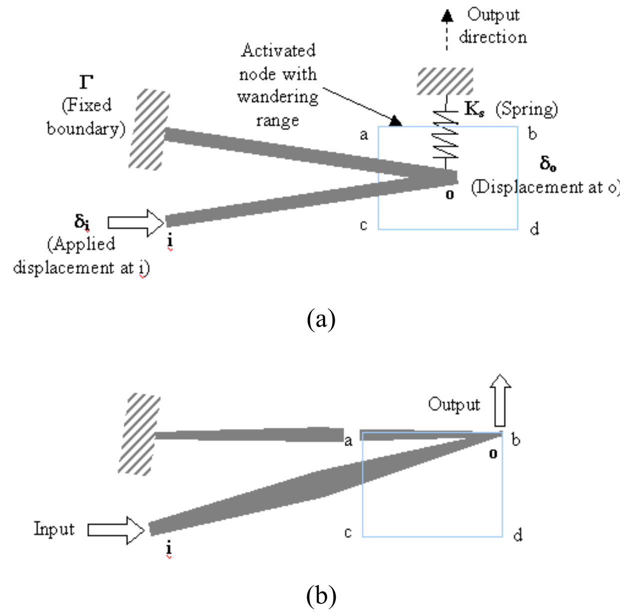


Fig. 2: Problem specifications and final result: (a) Problem specifications for size and shape optimization. (b) Optimized result showing tapered beam elements and changes in shape.

$$
\begin{aligned}
\max \quad & GA\left(=\frac{\delta_o}{\delta_i}\right) \\
\textit{subject to} \quad & Kd = f \\
& GA = GA^* \\
& \text{stress constraint} \\
& \Rightarrow \left(\begin{array}{c} \textit{if tension} + \textit{bending or compression} + \textit{bending} \geq 0 \\ \max\left(\left|\text{stress}^{top}\right|, \left|\text{stress}^{bottom}\right|\right) \leq \sigma_{allowable} \\ \textit{if tension} + \textit{bending or compression} + \textit{bending} < 0 \\ \left|stress\right| \leq \min\left(\sigma_{critical}, \sigma_{allowable}\right) \end{array} \right) \\
& f_{in} \leq f^* \\
& V \leq V^* \\
& A_{lower} \leq A \leq A_{upper} \\
& x_{lower}, y_{lower} \leq x, y \leq x_{upper}, y_{upper}
\end{aligned}
$$

δ_i and δ_o represent input and output displacements. Stress constraint is divided into two parts in order to avoid buckling when a link is under compressive stress. GA^*, f^*, V^* are the desired geometrical advantage, input force upper limit and the total volume limit, respectively. Here, the equality constraint of GA is included because a design with a specific GA is targeted in this formulation. If the program is not able to find a design which has a prescribed GA, the program will give an infeasible solution. “A” represents cross sectional area of each link, which is the size variable. “x and y” represents geometrical variables which defines the shape of mechanism by allowing nodes to change their positions within the specified range. The design process for size and shape optimization is shown in Figure 3.

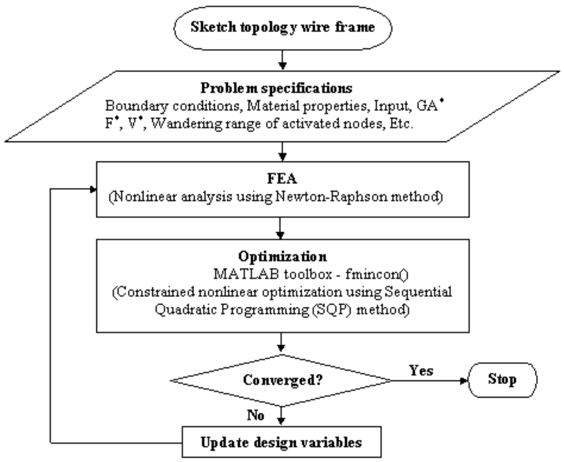


Fig. 3 Design procedure flowchart for size and shape optimization

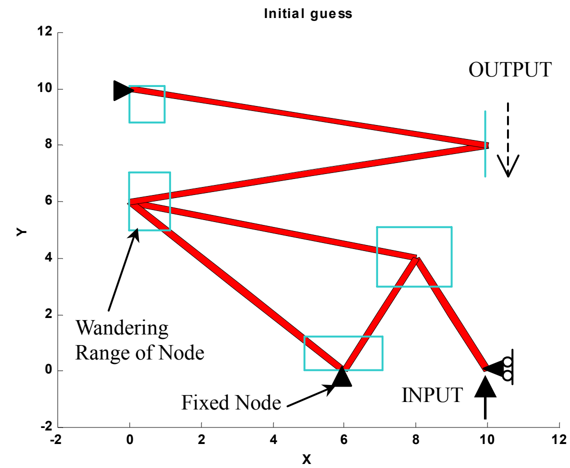
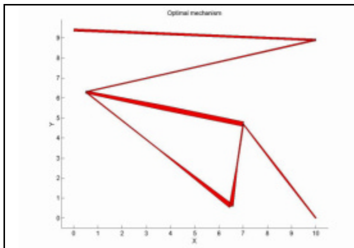


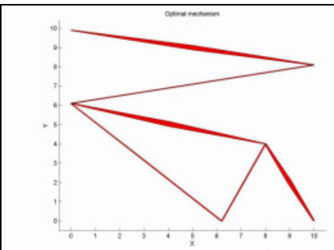
Fig. 4 Problem specifications for stroke amplifier design

DESIGN EXAMPLE

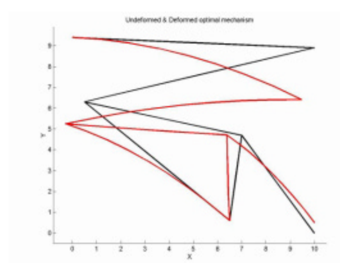
The size and shape optimization using non-linear beam elements is illustrated with an example problem where the goal is to maximize the geometric advantage of the mechanism. Results from linear formulations are then compared with those from non-linear formulations. A stroke amplifier is described with initial topology, size, shape and boundary conditions as shown in Figure 4. The direction of the desired output motion is out of phase (opposite to input direction) with the input displacement (0.5 units). In addition, a spring of stiffness 0.5 (unit force / unit displacement) is attached to the output node representing an external load acting against the output motion. The goal is to obtain five times displacement amplification (GA = 5) using both nonlinear and linear synthesis procedures under the same conditions. Finite equilibrium equations to verify displacement-force relations and the constrained nonlinear optimization equations are solved within MATLAB. The optimization function "fmincon()", which uses the sequential quadratic programming (SQP) method, is utilized for optimization and sensitivities are calculated using finite difference method. The size and shape optimization results are shown in the Figure 5a and 5b.



(a) Optimal mechanism from nonlinear synthesis

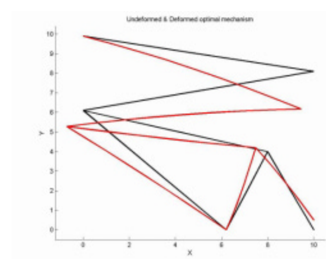


(c) Optimal mechanism from linear synthesis



Input Disp. = 0.5
Output Disp. = 2.5
GA = 5
Volume = 4.59

(b) Deformed shape of the mechanism from nonlinear synthesis results



Input Disp. = 0.5
Output Disp. = 1.9
GA = 3.8
Volume = 4.63

(d) Deformed shape of the mechanism from linear synthesis results (nonlinear analysis)

Fig. 5 Synthesis result; (a) & (b) Nonlinear; (c) & (d) Linear

NONLINEAR TO LINEAR SYNTHESIS COMPARISON

Both nonlinear and linear synthesis was done with the same set of specifications and results are shown in Figure 5a and 5b, respectively. In comparing the nonlinear and the linear results, nonlinear analysis was performed on both results as shown in Figure 5c and 5d.

The mechanism design from nonlinear synthesis attained a geometrical advantage (GA = output displacement / input displacement) of 5 as prescribed. The GA of mechanism design from linear synthesis, however, was calculated to be 3.8 (75 % of the expected GA) by the nonlinear analysis. Figure 6 shows the displacement histories of the output port in the negative y direction versus the input displacement. Figure 7 shows the GA versus the input displacement. Although, the linear formulation converged to a solution with GA=5 the actual GA turned out to be only 3.8 when non-linear analysis was used to verify the design. However, the GA of the mechanism from nonlinear theory reached 7 in the beginning and decreased to 5. This is the desired GA when a 0.5 unit input displacement is applied. Accordingly, linear synthesis is not appropriate for mechanisms operating in the large deformation range in which the force-displacement relationship is nonlinear.

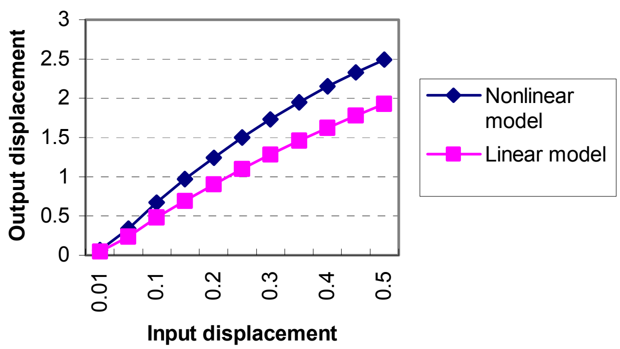


Fig. 6 Output – input displacement history

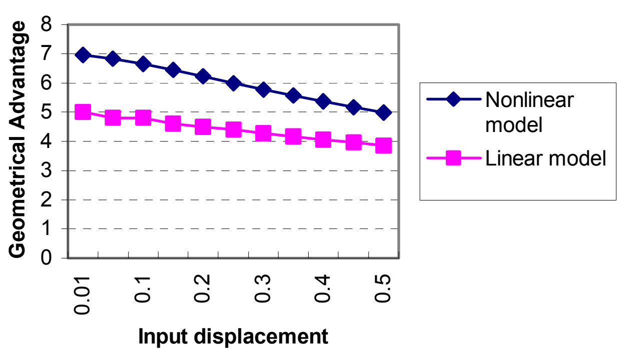


Fig. 7 Geometrical advantage history

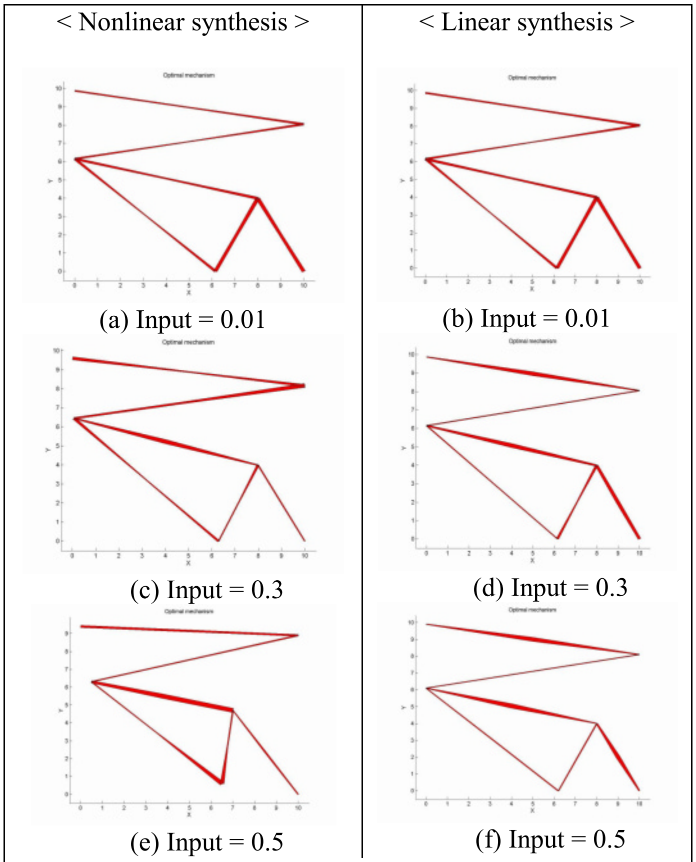
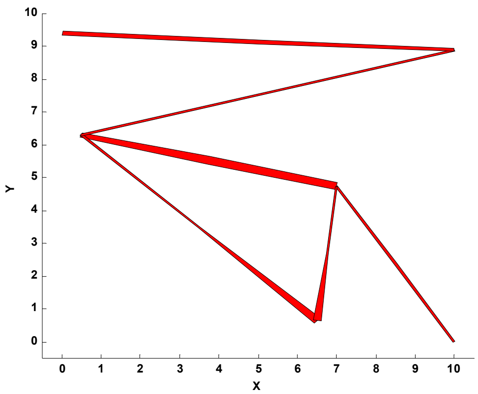
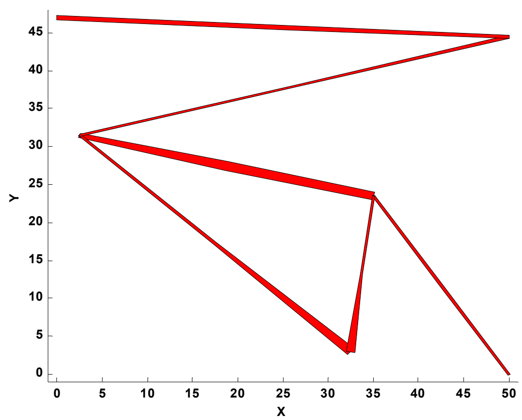


Fig. 8 Changes in optimized size and shape as the magnitude of input displacement is increased.

Figure 8 shows size and shape changes of optimized mechanisms based on nonlinear and linear synthesis according to different inputs (δ_i = 0.01, 0.3, and 0.5). For δ_i = 0.01, the size and shape from nonlinear and linear synthesis are identical. However, as the input displacement is increased, the differences between the two results are noticeable. When the input reached 0.5, only the nonlinear synthesis result showed changes in size and shape to reach a GA of 5 as explained earlier (Figure 8e). For the linear-synthesis based mechanism, the shape of Figure 8f is almost the same as that of Figure 8b, i.e., only the size of the links changed as a result of the stress change due to increased input. The reason is that the mechanism based on linear synthesis cannot capture the nonlinear effect because the sensitivity is constant regardless of the deformation range. This effect causes the loss of GA for linear-synthesis based mechanisms when the deformation exceeds the linear range. Therefore, nonlinear synthesis is advantageous for mechanisms that are required to undergo large deformations.



(a) Original model



(b) Scaled up model

Fig. 9 Original & 5x-scaled mechanisms

SCALE EFFECT OF MODELS

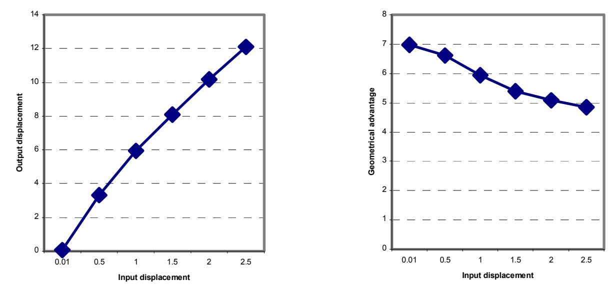
In this section, we present the effect of scaling on the behavior of an optimized mechanism. We employ non-linear formulation for optimization and compare force-displacement behavior of a given mechanism as it is scaled up or scaled down.

The optimal design in Figure 9a is simply enlarged without synthesis and compared with the original mechanism to understand the nonlinear behavior of scaled up mechanisms. For this experiment, every constituent such as size, shape, and displacement are multiplied by five except the material properties of the mechanism. The magnified mechanism is shown in Figure 9b.

The output displacement and geometrical advantage are outlined in Figure 10 as increasing the input up to 2.5, which is five times the original input displacement. Nonlinear analysis is also performed for the displacement calculation. The profiles of Figure 11 show that they are very close to that of the original mechanism illustrated in Figure 6 and 7 with the axes enlarged by a factor of five. This suggests that similar output behavior can be obtained regardless of size change while the size to deformation ratio remains the same as original problem.

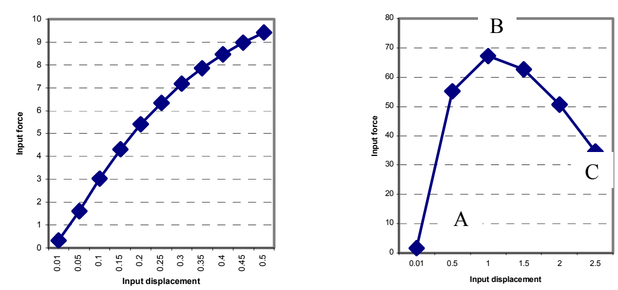
The input displacement and force plot results, however, do not match the original mechanisms results. The force-displacement relation of original mechanism is monotonic, though not linear (Figure 11a). For the magnified mechanism, the force-displacement relation is monotonically increasing within the range from the point A to point B (Figure 11b). Considering that compliant mechanisms are used within the monotonically increasing domain, the magnified mechanism is not valid beyond the point B where this domain ends.

Thus, it is possible to scale the original mechanism by enlarging the design domain proportionally and to use the result as a new device. However, the input cannot be increased proportionally due to the non-monotonic behavior of its force-displacement relation.



(a) Input-output displacement relationship (b) Geometrical advantage

Fig. 10 Displacements and geometrical advantage of scaled mechanism



(a) Original mechanism (b) Scaled up mechanism (five times)

Fig. 11 Force-displacement relationship (at input port) in original and scaled mechanism

At this time, using the mechanisms utilized in previous study (Figure 9), we now apply a 0.5 unit displacement input and attach springs with stiffness of 0.5 at the output port of both mechanisms. This investigation is for the case when a synthesized mechanism cannot generate enough output displacement but has no limits on design domain and actuation force. As a result of simple extension of the mechanism, keeping the size and shape ratio, the output displacement can be increased and the input-output displacements are almost linearly proportional (Figure 12). This linearity enables us to control the output without difficulty. These effects result from the extended linear range by enlarging the original mechanism. A drawback of this extended mechanism, however, is that the required input force is greater than the force used for original mechanisms due to the increased stiffness of the enlarged mechanisms. Also, GA is higher than the desired value. If enough power is available and larger GA is permissible, this simple extension of the original mechanism is a viable alternative to synthesizing a new mechanism.

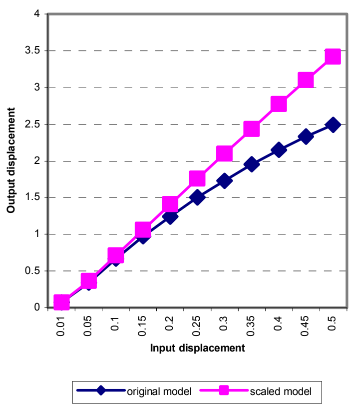
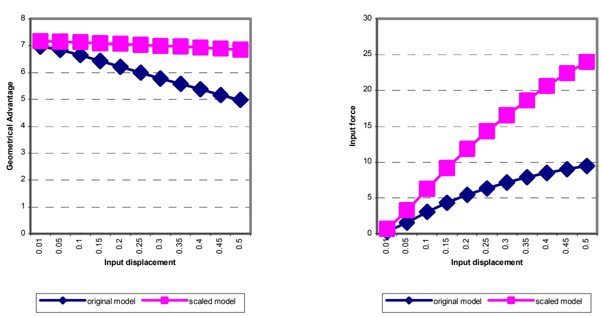


Fig. 12 Input-output displacement comparison between original and scaled mechanism



(a) Geometrical advantage

(b) Force-displacement relation

Fig. 13 Comparison between original and the scaled mechanism (nonlinear analysis)

CONCLUSION

In this paper, size and shape synthesis methodology considering geometrical nonlinearity has been demonstrated using tapered beam elements. The stroke amplifier example shows that the GA of the mechanism after linear synthesis reached only 75% of that of nonlinear synthesis result. The size and shape of the designs change as the applied force or displacement is changed. The paper also demonstrated that the scaling laws apply to compliant mechanism designs with non-monotonic behavior in force-displacement relationship.

REFERENCES

[1] Joo, J.; Kota, S.; Kikuchi, N. 2000: Nonlinear Synthesis of Compliant Mechanisms: Topology Design, *Proc. of 2000 ASME Biannual Mechanism Design Conference*, DETC2000/MECH-14141

[2] Pedersen, C.B.W.; Buhl, T.; Sigmund, O. 1999: Topology Synthesis of Large-Displacement Compliant Mechanisms. *Proc. Of 1999 ASME Engineering Technical Conference*. DETC99/DAC-8554, CD-ROM

[3] Saxena, A.; Ananthasuresh, G. K. 1999: Towards the Design of Compliant Continuum Topologies with Geometric Nonlinearity. *Proc. Of 1999 ASME Engineering Technical Conference*. DETC99/DAC-8578, CD-ROM

[4] Bruns, D.A.; Tortorelli, D.A. 1998: Topology Optimization of Geometrically Nonlinear Structures and Compliant Mechanisms*, 7th symposium on Multidisplinary Analysis and Optimization*, AIAA/USAF/NASA/ISSMO, AIAA-98-4950, 1874-1882

[5] Hetrick, J. A. and Kota, S., 1999, An Energy Formulation for Parametric Size and Shape Optimization of Compliant Mechanisms, *J of Mechanical Design* 121: 229-234

[6] Joo, J.; Kota, S.; Kikuchi, N. 2000: Topological Synthesis of Compliant Mechanisms Using Linear Beam Elements, *Mechanics of Structures and Machines*. 28 No.4, 245-280