The 14th IFToMM World Congress, Taipei, Taiwan, October 25-30, 2015
DOI Number: 10.6567/IFToMM.14TH.WC.OS2.048

An Algorithm for the Automatic Sketching of Generalized Kinematic Chains

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Abstract: *In the process of conceptual design or structural synthesis of mechanisms, an important step is to synthesize and sketch various atlases of kinematic chains or graphs. An algorithm for sketching generalized kinematic chains is proposed based on graph theory and link adjacency matrices. And, a computer program is developed for such a purpose.*

Keywords: generalized kinematic chains, link adjacency matrix, contracted graphs, conceptual design of mechanisms

I. Introduction

In early studies, kinematic chains were simply enumerated based on intuition or inspiration [1-3]. Started in the 1960s, various researchers demonstrated the feasibility of synthesizing the kinematic structure of mechanisms with closed chains by means of more formal methods such as Franke's notation [4-6], graph theory [7-9], Baranov Trusses concept [10-13], and so on. However, such methods cannot guarantee to generate all kinematic chains or to eliminate isomorphic chains. Moreover, being manual methods, they are laborious and time-consuming.

Olson et al. [14] presented a computer-aided synthesis process based on graph theory and vertex-edge incidence matrices for sketching the closed-loop kinematic chains with simple joints. Yan and Hwang [15] proposed an algorithm and developed a computer program to sketch kinematic chains automatically for the minimization of the number of crossing links based on the concepts of basic contracted kinematic chains. They further presented an approach along with a computer program for the number synthesis of kinematic chains with up to 12 links and without isomorphic chains based on contracted link adjacency matrices and permutation groups [16]. Hwang and Hwang [17] proposed the contracted link adjacency matrices to represent the topological structure of the kinematic chains, and an approach was presented for the computer-aided structural synthesis of planar kinematic chains with simple joints. Moreover, a computer program was developed to automatically generate the kinematic chains. The numbers of kinematic chains with up to thirteen links were listed. Belfiore and Pennestri [18] presented a sketching procedure for obtaining kinematic chains with up to 10 links based on graph embedding methods. Tuttle [19] developed a computer program to generate and enumerate planar, non-fractionated, pin-jointed closed kinematic chains with 2-6 independent loops based on the finite symmetry group theory. Ding et al. [20-23] developed a fully-automatic computer program to synthesize the kinematic chains with different DOFs based on the characteristic representation code, the rigid sub-chain detection and isomorphism identification method. Hsieh et al. [24] proposed a procedure for the structural synthesis of generalized kinematic chains with simple joints and without any cut-links (or cut-joints) by utilizing the contracted link adjacency matrices and the multiple link adjacency matrices.

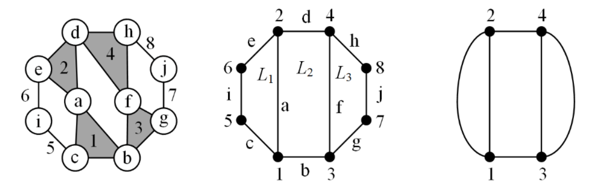
Yan and Chiu [25, 26] proposed algorithms for the construction of generalized kinematic chains without cut-links and nonplanar chains based on graph theory, multiple link adjacency matrices, and concept of Kuratowski graphs. Accordingly, the present study extends the method proposed in [25, 26] and implements it into a computer code to sketch the various atlases of generalized kinematic chains with simple joints.

II. Terminology and Definitions

For the arguments presented in this paper, the following terminology and definitions are needed.

A. LAM and MLAM

The *LAM* (link adjacency matrix) of a generalized kinematic chain with N_L links and N_J joints is an $N_L \times N_L$ matrix with its elements e_{ij}=1 if link i is adjacent to link j, and e_{ij}=0 otherwise. For the (8, 10) generalized kinematic chain shown in Fig. 1(a), its *LAM*, is:



(a) Generalized K. C. (b) Planar block (c) Contracted graph

Fig. 1 An (8, 10) generalized kinematic chain, and its corresponding planar block and contracted graph

$$LAM_{(8,10)} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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The *MLAM* (multiple link adjacency matrix) of a generalized kinematic chain with N_m multiple links and N_J joints is an $N_m \times N_m$ matrix with its elements $e_{ij}=n_1n_2n_3...n_x$ if multiple link i is adjacent to multiple link j with n series joints and x is the number of different types of contracted links, and $e_{ij}=0$ otherwise. Therefore, the dimension of the *MLAM* depends on the number of multiple links. For the (8, 10) generalized kinematic chin shown in Fig. 1(a), its *MLAM* is:

$$MLAM = \begin{bmatrix} 0 & 31 & 1 & 0 \\ 31 & 0 & 0 & 1 \\ 1 & 0 & 0 & 31 \\ 0 & 1 & 31 & 0 \end{bmatrix}$$

in which elements $e_{12}=e_{21}=31$ mean that one contracted link with three series joints (S_{J3}) and one contracted link with one series joint (S_{J1}) are adjacent to two ternary links.

B. Link Assortments and Multiple Link Assortments

The A_L (link assortment) of a generalized kinematic chain is the type and the number of links in the chain. It is a set of numbers consisting of the numbers of binary links (N_{L2}), ternary links (N_{L3}), quaternary links (N_{L4}), etc., and is expressed as: $A_L=[N_{L2}/N_{L3}/N_{L4}/…/N_{Lm}/…]$. The A_{ML} (multiple link assortments) of a generalized kinematic chain is the number and the type of multiple links in the chain. It is a set of numbers consisting of the numbers of ternary links (N_{L3}), quaternary links (N_{L4}), …, n links (N_{Lm}), and son on, and is expressed as: $A_{ML}=[N_{L3}/N_{L4}/…/N_{Lm}/…]$. The atlas of various generalized kinematic chains can be synthesized by assembling the corresponding link assortments and multiple link assortments.

The link assortments and multiple link assortments can be obtained by solving the following two equations:

$$N_{L2} + N_{L3} + ... + N_{Ln} + ... + N_{Lm} = N_L \quad (1)$$

$$2N_{L2} + 3N_{L3} + ... + nN_{Ln} + ... + mN_{Lm} = 2N_J \quad (2)$$

where m is the maximum number of joints incident to a link. Moreover, the number of joints N_J is constrained by the following equation:

$$N_L \le N_J \le N_L(N_L - 1)/2 \quad (3)$$

The maximum value, m_{max}, can be obtained by the following expression:

$$m_{\max} = \begin{cases} N_J - N_L + 2 & \text{if } N_L \le N_J \le 2N_L - 3 \\ N_L - 1 & \text{if } 2N_L - 3 \le N_J \le N_L(N_L-1)/2 \end{cases} \quad (4)$$

According to Eqs. (1)-(4), all possible multiple link assortments of the generalized kinematic chains can be obtained.

For the (8, 10) generalized kinematic chain shown in Fig. 1(a), $N_L= 8$ and $N_J= 10$, based on Eq. (4), m_{max} is:

$$m_{\max} = N_J - N_L + 2 = 4$$

Therefore, Eqs. (1) and (2) become:

$$N_{L2} + N_{L3} + N_{L4} = 8$$

$$2N_{L2} + 3N_{L3} + 4N_{L4} = 20$$

By solving these two equations, the link assortments are [4/4/0], [5/2/1], [6/0/2], and the multiple link assortment are [4/0], [2/1], [0/2].

C. Blocks and Planar Blocks

A block is a graph and a maximal non-separable subgraph which is connected, nontrivial, and without any cut-vertex. It is also called a 2-connected graph or non-separable graph. In addition, a planar block is a block which can be drawn in the plane with no edge crossings. A planar block with i vertices and j edges is called a (i, j) planar block. For example, the graph shown in Fig. 1(b) is an (8, 10) planar block.

D. Loops

In graph theory, a loop, L, is a closed walk in which the initial and end vertices are the same. For any graphs, G, with V vertices and E edge, the number of loops can be obtained by solving the following equation:

$$L = E - V + 1 \quad (5)$$

For the (8, 10) planar block shown in Fig. 1(b), Loop 1 (L_1) consists of vertices 1, 5, 6, 2, Loop 2 (L_2)consists of vertices 1, 2, 4, 3, and Loop 3 (L_3) consists of vertices 3, 4, 8, 7.

E. Contracted Graphs and Basic Contracted Graphs

A contracted graph is a graph comprising only vertices with more than two degrees (binary vertices) and is obtained by contracting all binary vertices until no binary vertices exist in the graph. For example, the (8, 10) planar block shown in Fig. 1(b), its corresponding contracted graph is shown in Fig. 1(c).

A basic contracted graph is a graph which consists of multiple vertices (links), i.e., ternary vertices (links), quaternary vertices (links), and so forth. The basic contracted graphs can be obtained by contracting binary vertices (links) until no binary vertices (links). The atlas of basic contracted graphs with up to five loops were presented by Tempea [27], then Yan and Hwang [2] obtained 19 basic contracted graphs with up to five loops. Based on the numbers of loops, Eq. (5), and its multiple link assortments in Section 2.2, the basic contracted graphs are constructed and classified. Therefore, the basic contracted graphs with two to four loops are presented in Fig. 2 and those with five loops are in References [27, 2]. This is the data bank for the sketching of the generalized kinematic chains and planar blocks. For example, the (8, 10) generalized kinematic chain as shown in Fig. 1(a), the corresponding basic contracted graph is shown in Fig. 2(c2). It is synthesized by contracting binary vertices 5, 6 and 7, 8.

F. Line Graphs and Hypergraphs

A hypergraph is a pair of sets (v, y) in which v is a set of elements called vertices and y is a nonempty set of elements called hyperedges. Furthermore, it is a graph in which hyperedges may connect more than two vertices and no two hyperedges consist of the same set of vertices. Hypergraphs are drawn by representing each hyperedge as a closed curve containing its vertices.

For a graph G, its line graph G_L has the edges of graph G as its vertex set. The two vertices V_1 and V_2 of G_L are adjacent when each edge of graph G is the set consisting of the two vertices that it joins. The intersection $V_1 \cap V_2$ is a singleton. In other words, vertices V_1 and V_2 of G_L are joined by an edge in G_L if edges E_1 and E_2 of G are incident with just one common vertex of G. For the (8, 10) planar block shown in Fig. 1(b), its corresponding hypergraph and line graph are shown in Figs. 3(a) and (b), respectively.

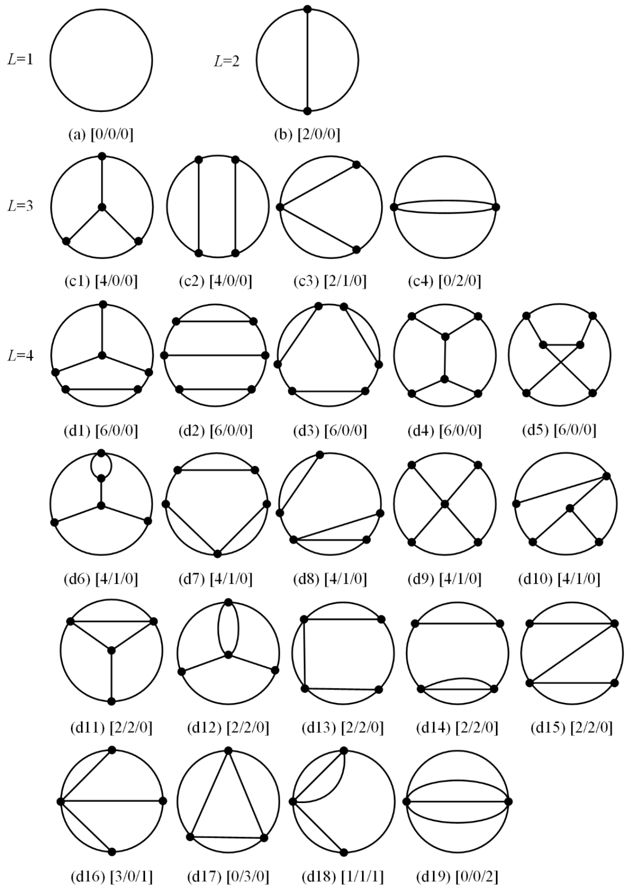


Fig. 2 Basic contracted graphs with up to four loops

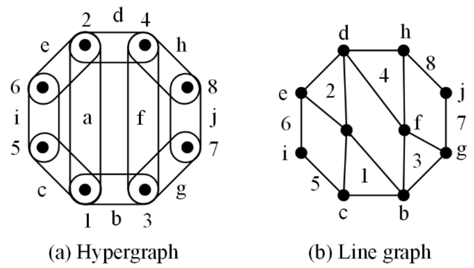


Fig. 3 An (8, 10) hypergraph and line graph

III. Synthesis of *LAM*

In order to sketch the generalized kinematic chains, an algorithm for the synthesis of link adjacency matrix (*LAM*) is proposed. The *LAM* can be obtained by transforming the multiple link adjacency matrix (*MLAM*). Besides, the *MLAM* synthesis procedure and constraints are proposed by Yan and Chiu [26]. The algorithm for the synthesis of *LAM* is shown in Fig. 4 and the main steps are described as follows:

Step 1. Input an MLAM and set initial values Dim and NewDim

Since the *MLAM* and *LAM* are both symmetric matrices, the *LAM* can be synthesized by calculating the elements of either the upper triangular matrix or the lower triangular matrix. Each element of the upper triangular *MLAM* should be inputted. In addition, the value *Dim* is the dimension of the *MLAM* and is a constant. The value *NewDim* is a variable. The meaning of the value *NewDim* is to expand the dimension of the matrix. The element e_{ij} of *MLAM* is imported to perform the next step.

Step 2. Calculate LAM[i, j] and LAM[j, i]

The values x and e_{ij} can be obtained based on the following equations:

$$x = e_{ij} - e_{ij}/10 \times 10 \tag{6}$$

$$e_{ij} = e_{ij}/10 \tag{7}$$

where e_{ij} is the element of the *MLAM*[*i, j*] and the value $e_{ij}/10$ must be an integer. The purpose of Eq. (6) is to save the element of the last digit, i.e., the one before the decimal point. The meaning of the value x is to find out the adjacency relationship of the contracted links. The purpose of Eq. (7) is to save the remaining digits. It means to save the other types of contracted links in the chain. If the value x=0, i.e., there are no relationship between link i and link j; then, go back to Step 1 and perform another element. If the value x=1, i.e., link i is adjacent to link j. Then, *LAM*[*i*, *j*]=*LAM*[*j*, *i*]=1. Otherwise, go to Step 3.

Step 3. Calculate LAM[NewDim+1, i] and LAM[i, NewDim+1]

Since the value x≠1, *LAM*[*NewDim*+1, *i*]=*LAM*[*i*, *NewDim*+1]=1. If the value, x-2>0 and value k=1, go to Step 4. Otherwise, go to Step 5.

Step 4. Calculate LAM[NewDim+k, NewDim+k+1] and LAM[NewDim+k+1, NewDim+k]

If the value k≤x-2, *LAM*[*NewDim*+*k*, *NewDim*+*k*+1]= *LAM*[*NewDim*+*k*+1, *NewDim*+*k*]=1. Then, the new value k=k+1, and repeat this step again. If no, go to Step 5.

Step 5. Calculate LAM[NewDim+x-1, j] and LAM[j, NewDim+x-1]

Since the value x-2≤0 or k>x-2, *LAM*[*NewDim*+*x*-1, *j*]=*LAM*[*j*, *NewDim*+*x*-1]=1. Then, the new value *NewDim*=*NewDim*+*x*-1 and go back to Step 2. Once all elements are determined, the *LAM* can be obtained and saved.

The purpose of Steps 3 and 5 are to obtain the relationship between the binary links and multiple links. The purpose of Step 4 is to obtain the relationship between two binary links. Through *LAM* synthesis algorithm, the *LAM* can be synthesized and obtained.

IV. Generalized Kinematic Chains and Planar Blocks

Since the early 1960s, graph theory has been applied to the number and structural synthesis and analysis of various types of chains and mechanisms. Based on Reference [28], a planar block can be transformed into its corresponding generalized kinematic chains by representing the vertices and edges of the planar blocks with links and joints, respectively, in which two links in the chain are adjacent whenever the corresponding vertices in the graph are adjacent. For a given planar block, the following process describes how to construct the corresponding generalized kinematic chain based on graph theory:

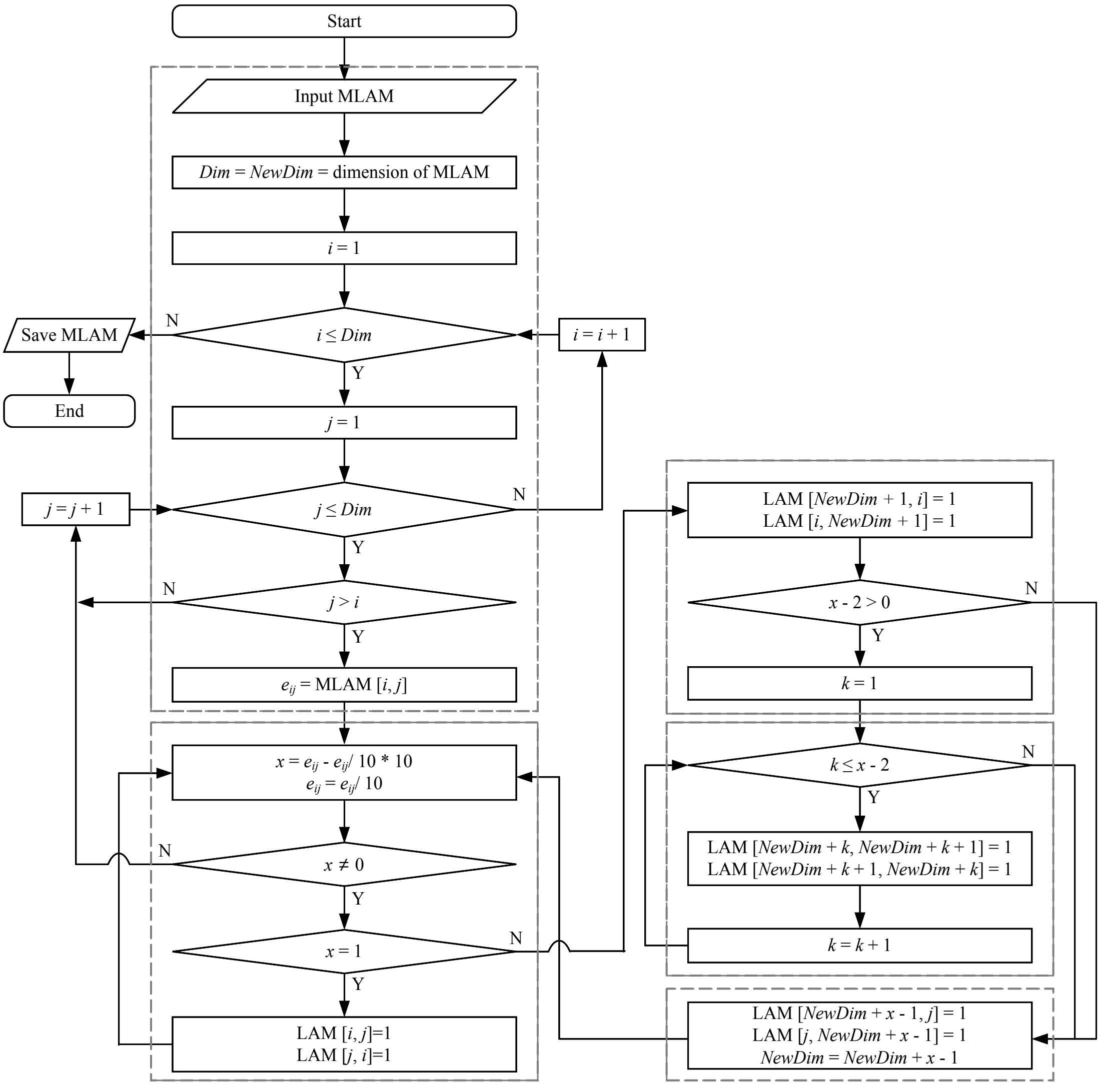


Fig. 4 Synthesis algorithm of LAMs

Step 1. For each vertex, list those edges incident with the vertex.

Step 2. Construct the corresponding line graphs G_L described in Section II-F.

Step 3. Replace each vertex of G_L with a small circle and replace each complete subgraph of G_L which is determined by a vertex of a planar block of degree at least three by a shaded polygon. This is done by removing the interior edges to obtain a perimeter polygon and then shading the interior of this polygon.

Each planar block has a single associated generalized kinematic chain and every generalized kinematic chain constructs a single planar block.

V. Sketching Algorithm

An algorithm for sketching nice looking generalized kinematic chins is proposed based on its corresponding *LAM* and graph theory, as shown in Fig. 5, and each step is illustrated with an example.

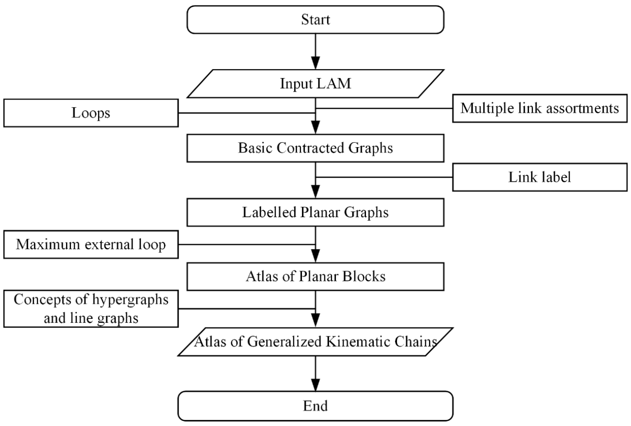


Fig. 5 Sketching algorithm

Step 1. Link adjacency matrix

The *LAM* of a generalized kinematic chain is the input data. Based on the *LAM* synthesis algorithm in previous section, the *LAM* can be determined.

The (8, 10) generalized kinematic chain is used as an illustrative example.

Step 2. Basic contracted graphs

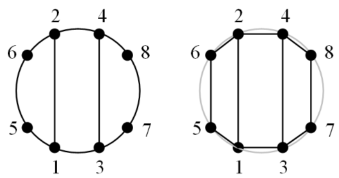
Based on the multiple link assortments and the number of loops, the basic contracted graphs can be identified from Fig. 2. Based on Eq. (5), the number of loops is three. Since the multiple link assortment A_{ML}=[4/0], there are four ternary vertices in the graph. Therefore, the basic contracted graph with three loops and four ternary vertices can be obtained from Figs. 2(c1) and (c2).

Step 3. Labelled graphs

Based on the *LAM*, binary vertices need to be added to the edges of its basic contracted graph. Then, vertices *x* and *y* are connected by an edge which are adjacent to each other based on its *LAM*. Thus, the labelled planar graphs can be obtained.

Based on the *LAM*, each ternary vertex is adjacent to two ternary vertices. Therefore, the basic contracted graph shown in Fig. 2(c2) is selected to synthesize the (8, 10) generalized kinematic chains.

The ternary vertices can be labelled as “1”, “2”, “3”, and “4” in the basic contracted graphs as shown in Fig. 6(a). Then, binary vertices must be added in the graph shown in Fig. 6(b). Binary vertices 5 and 6 should be added between vertices 1, 2, and connected the vertices 1, 5, 6, 2 by edges based on the $LAM_{(8,\ 10)}$. Binary vertices 7 and 8 should be added between vertices 3, 4, and connected the vertices 3, 7, 8, 4 by edges based on the $LAM_{(8,\ 10)}$. Then, the labelled graph with eight vertices and ten edges can be obtained as shown in Fig. 6(b).



(a) (b)

Fig 6. Labelled graphs with eight vertices and ten edges

Step 4. Atlas of planar blocks

Based on the labelled graphs obtained in Step 3, the maximum external loop can be determined. The outer circle should be deleted. Then, the planar blocks should be redrawn and the atlas of planar blocks can be obtained.

For the (8, 10) labelled graph shown in Fig. 6(b), the maximum external loop consists of vertices 1, 5, 6, 2, 4, 8, 7, 3. Since there are eight vertices in the external loop, this is an octagon. Therefore, the graph is redrawn, and the (8, 10) planar block can be obtained as shown in Fig. 1(b).

Step 5. Atlas of generalized kinematic chains

Each planar block can be transformed into a single generalized kinematic chain based on the atlas of planar blocks obtained in Step 4 and graph theory. In addition, a generalized kinematic chain can be transformed from its corresponding planar block based on the proposed method in Section 4.

For the (8, 10) planar block shown in Fig. 1(b), the generalized kinematic chain can be obtained as shown in Fig. 1(a).

Based on the proposed sketching algorithm, the aesthetic characteristics of generalized kinematic chains and planar blocks with no link or edge crossings can be sketched and obtained.

VI. Computer Program

The sketching algorithm and proposed algorithms in References [25, 26] are implemented into a computer program in order to facilitate the automatic sketching of generalized kinematic chains and planar blocks. This computer program is developed by using Microsoft Visual Studio 2012 with programming language C# running on the Windows Platform. The program is executed on a PC with an Intel ® Core™ i7-3770 CPU 3.40GHz processor and 8GB RAM. Given only the numbers of links and joints as the input data, the program automatically computes the link assortments and contracted link assortments, and sketch generalized kinematic chains and planar blocks. Furthermore, the computer program has up to 20 functions in the user interface as shown in Fig. 7. The atlases of generalized kinematic chains and planar blocks are shown in the middle of interface. An example is provided to illustrate the computer program for the construction of generalized kinematic chains with different numbers of links and joints.

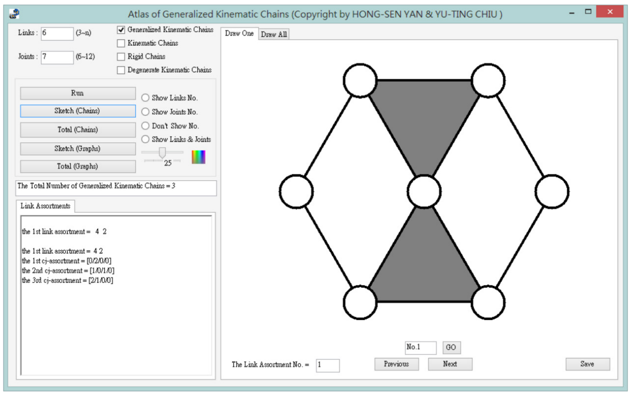


Fig. 7 User interface of the computer program

Example: Atlas of (8, 10) generalized kinematic chains

When the parameters are given as Links=8 and Joints=10, the link assortments are A_L=[4/4/0], [5/2/1], and [6/0/2]. The atlas of (8, 10) generalized kinematic chains and can be obtained shown in Fig. 8. Therefore, the atlases of (8, 10) generalized kinematic chains can be synthesized and sketched automatically according to the developed computer program. And, the number of the atlases of (8, 10) generalized kinematic chains is 40.

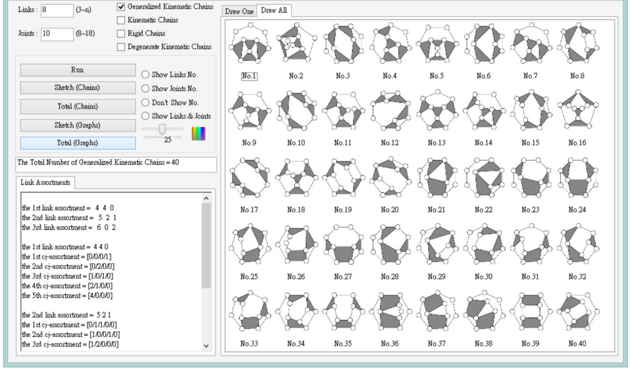


Fig. 8 Atlas of (8, 10) generalized kinematic chains

VII. Conclusions

An algorithm for the sketching of generalized kinematic chains is proposed based on the concepts of the link adjacency matrices and the basic contracted graphs. A computer program is developed for automating the synthesis and sketching of various atlases of generalized kinematic chains and planar blocks based on the proposed algorithms. Through the computer program, generalized kinematic chains with simple joints and no crossing links are obtained. An important feature for adjusting to have a nice looking generalized kinematic chain is achieved based on the identification of the maximum external loop to turn inside out of the chain. As a result, the synthesized atlases of generalized kinematic chains with required numbers of links and joints provide mechanism designers the necessary data bank for the generation of all possible topological structures in the conceptual stage of mechanism design.

Acknowledgements

The authors are grateful to the Ministry of Science and Technology (Taiwan) for supporting this research under grant MOST 103-2221-E-006-042.

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